



SPATIOTEMPORAL ANALYSIS OF SKELETAL MUSCLE TEMPERATURE UNDER DYNAMIC CONTRACTIONS

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Abstract

This study extends Pennes' bioheat equation to capture the thermal behavior of skeletal muscle undergoing periodic contraction and relaxation. By adopting a moving (Lagrangian) coordinate system and employing Fourier expansion, the model effectively predicts how heat is generated, conducted, and dissipated in actively contracting muscles. It provides a mechanistic explanation for why exercising muscles warm up, how they approach thermal steady state, and how convective cooling at the skin surface influences overall temperature distribution. Using this framework, the three-dimensional temperature profile within skeletal muscle was simulated, starting from a uniform dimensionless temperature representing the relative difference between arterial blood and ambient temperatures. As the simulation progressed, pronounced temperature gradients emerged along the muscle thickness due to the interplay of metabolic heat generation, blood perfusion, and surface heat dissipation. Periodic contractions induced oscillatory variations in local temperature, emphasizing the role of dynamic mechanical deformation on thermal responses. Notably, deeper muscle regions exhibited higher temperature elevations than superficial layers, in agreement with physiological expectations. Overall, this computational approach provides a robust framework for investigating spatiotemporal thermal variations in skeletal muscle, offering valuable insights for applications in thermoregulation studies, sports physiology, thermal therapy, and biomedical device design.

Keywords: Skeletal muscle thermoregulation, Pennes' bioheat equation, Three-dimensional temperature distribution, Periodic muscle contraction, Metabolic heat generation, Blood perfusion, thermo-mechanical modeling, Dimensionless temperature, Spatiotemporal thermal variations, Soft tissue heat transfer.

INTRODUCTION

The thermal behavior of skeletal muscle during physical activity is a critical aspect of human physiology, as it underpins tissue metabolism, perfusion, and thermoregulation. Skeletal muscles, being highly metabolically active tissues, generate substantial amounts of heat during contraction due to biochemical reactions involved in ATP hydrolysis and mechanical work [1,2]. This metabolically generated heat is transported through the tissue by conduction and convective mechanisms associated with blood perfusion, while a portion is dissipated to the surrounding environment through the skin and subcutaneous layers via convective and radiative heat transfer [3,4]. Understanding these processes is not only fundamental from a physiological perspective but also essential in biomedical applications such as hyperthermia therapy, cryotherapy, prosthetic design, and the development of wearable thermal sensors [5,6]. Modeling the temperature distribution within skeletal muscle is inherently complex due to the dynamic nature of muscle contraction and the interplay between metabolic heat generation, blood flow, and thermal boundary conditions. Traditional heat transfer models, such as Fourier's law of conduction, fail to fully capture the contributions of perfusion and active metabolic processes. To address this, Pennes' bioheat transfer equation has been widely employed as a robust framework for simulating tissue temperature dynamics, as it incorporates

the effects of blood perfusion, metabolic heat generation, and thermal conduction within biological tissues [7]. Numerous studies have applied Pennes' model to understand thermal gradients in muscles during rest, exercise, and pathological conditions, demonstrating its utility in capturing physiologically relevant temperature distributions [8,9].

In addition to metabolic heat, mechanical activity within skeletal muscles—particularly periodic contractions—introduces spatiotemporal variations in tissue deformation, blood flow, and local heat generation. These mechanical effects can modulate perfusion and influence localized temperature gradients, highlighting the need for a coupled thermo-mechanical modeling approach [10,11]. Recent computational studies have employed Lagrangian frameworks and finite element simulations to incorporate the dynamic effects of periodic contractions, enabling detailed analysis of how mechanical deformation interacts with metabolic and perfusion-driven heat transfer [12,13]. In this study, we investigate the three-dimensional temperature distribution within skeletal muscle subjected to periodic contractions.

The system is initialized with a uniform dimensionless temperature profile, representing the relative difference between arterial blood and ambient temperatures. The simulation captures the evolution of temperature gradients resulting from the interaction of metabolic heat, blood perfusion, and surface heat dissipation. Moreover, the effects of oscillatory muscle contractions on local temperature variations are examined, providing insights into the complex thermoregulation mechanisms in active skeletal muscles. These findings not only confirm physiological expectations of non-uniform heating in deeper muscle regions but also validate the computational framework for studying coupled thermo-mechanical behavior in soft tissues [14,15]. By integrating metabolic, perfusive, and mechanical effects, this work contributes to a deeper understanding of skeletal muscle thermodynamics, which is essential for optimizing therapeutic interventions, designing biomedical devices, and advancing computational models of human tissue physiology.

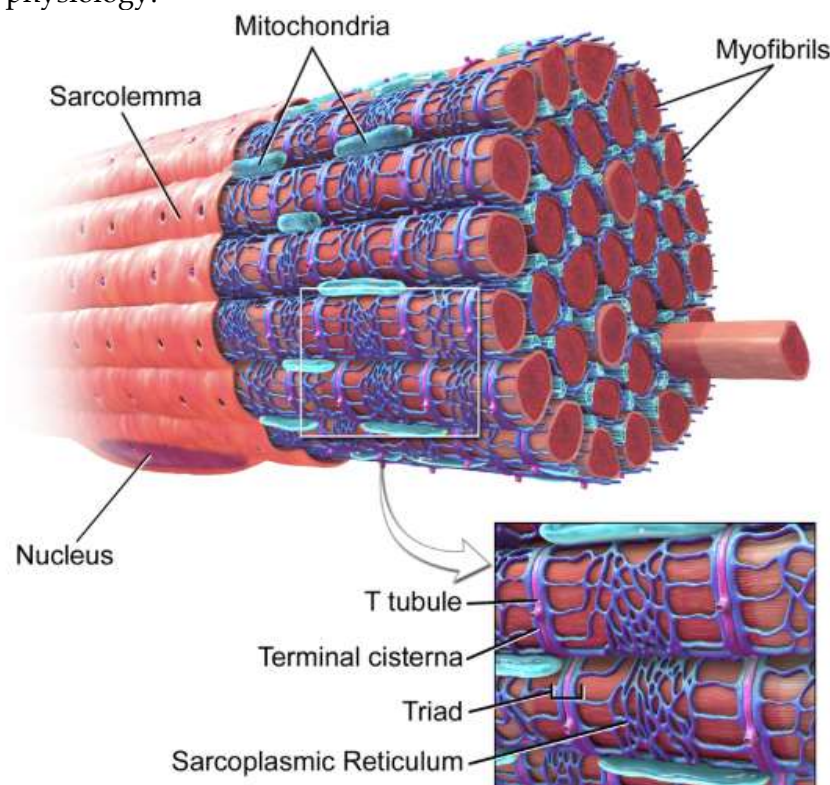


Figure (1): Skeletal Muscle

Formulation of the problem: The temperature distribution in skeletal muscle and skin is governed by the Pennes' bioheat transfer equation:

$$\nabla(k\nabla T) + \dot{m}_b c_b (T_0 - T) + S_m = \rho c \frac{\partial T}{\partial t} \quad (1)$$

where: T is the temperature ($^{\circ}\text{C}$), ρ is the density of tissue (kg/m^3), c is the specific heat capacity of tissue ($\text{J}/\text{kg}\cdot^{\circ}\text{C}$), k is the thermal conductivity of tissue ($\text{W}/\text{m}\cdot^{\circ}\text{C}$), c_b is the specific heat capacity of blood ($\text{J}/\text{kg}\cdot^{\circ}\text{C}$). The mass flow rate of blood is represented by $\dot{m}_b$, T_0 is the arterial blood temperature ($^{\circ}\text{C}$), s_m is the metabolic heat generation rate (W/m^3). The goal of this work is to look at the situation when muscle lengthening has a linear time dependency [16].

The skeletal muscle used in this model undergoes regular, isotonic contractions, and its volume may be assumed to remain constant in accordance with physiological realities. The skeletal muscle is conceptualised as a rectangular plate, with the following description of motion:

$$x = s_1(1 - A\cos\omega t); \quad (2)$$

$$y = s_2; \quad (3)$$

$$z = \frac{s_3}{(1 - A\cos\omega t)}; \quad (4)$$

It is defined as the muscle thickness (initial dimension) at rest, denoted by $A=0$. Equ. (2) (3) and (4) uses the following terms: ω represents the frequency of muscle contractions, A is a constant reflecting the maximum extent of muscle shortening, and s_1 , s_2 , and s_3 are constants that do not vary with time [17]. The significance of these constants will be clarified as the discussion unfolds. In this context, x and y refer to standard Cartesian coordinates, and t denotes time. The maximum contraction of smooth muscle can reach 60–75% of its initial length, which is greater than the maximum contraction observed in skeletal muscle. Consequently, $A < 0.5$ across the entire muscle surface, except in the area directly beneath the skin where heat exchange with the external environment occurs.

The boundary conditions are as follows:

At the inner boundary of the tissue, it is assumed that there is no heat flux, this implies that no thermal energy is being transferred across this boundary, reflecting an insulated or symmetry condition within the muscle. At the outer boundary, corresponding to the skin surface, the tissue exchanges heat with the surrounding environment through convection. This convective heat loss is governed by the temperature difference between the skin and the ambient environment, allowing thermal energy to dissipate from the tissue to the surroundings.

$$\begin{aligned} \frac{\partial T(t,0)}{\partial z} &= 0; \\ \frac{-\partial T(t,l(1-A\cos\omega t))}{\partial z} &= \alpha(T - T_a) \end{aligned} \quad (5)$$

where T_a represents the ambient temperature and α is the heat transfer coefficient, which is assumed to be constant for the given model. However, the intricate and variable heat insulating properties and temperature distribution immediately beneath the muscle, particularly within the hypodermic adipose tissue, must be considered at the skin's boundary. Therefore, the two-layer model and the associated boundary conditions are crucial for accurate analysis.

Non-Dimensionalisation: New variables are introduced to simplify equations:

$$\begin{aligned} \theta &= \frac{T - T_a}{T_0}; \\ \sigma &= \frac{s_3}{l} = \frac{z(1 - A\cos\omega t)}{l}; \\ q &= \frac{s_m l^2}{k T_0}; \beta = \frac{\omega c_b \rho_b l^2}{k}; \\ \tau &= \frac{kt}{c_b \rho_b l^2}; g = \frac{\dot{m}_b c_b l^2}{k} \end{aligned} \quad (6)$$

Finally, we get

$$\frac{\partial \theta}{\partial \tau} = (1 - A\cos\beta\tau)^2 \frac{\partial^2 \theta}{\partial \sigma^2} + g \left[\left(1 - \frac{T_a}{T_0}\right) - \theta \right] + q \quad (7)$$

using the conditional mechanism

$$\begin{aligned} \frac{\partial \theta(\tau,1)}{\partial \sigma} &= \frac{-\alpha l \theta}{k(1 - A\cos\beta\tau)} \\ \frac{\partial \theta(\tau,1)}{\partial \tau} &= 0 \end{aligned} \quad (8)$$

The initial muscle temperature should be the same as the blood temperature

$$\theta(0, \sigma) = 1 - \frac{T_a}{T_0} \quad (9)$$

Variable substitution (7) involves transforming Eulerian variables into Lagrangian variables.

The Lagrangian approach is more practical in this context due to body deformation and boundary mobility. A coordinate system, which moves with the body's deformation and remains in direct contact with it, is used. The variables s_1 , s_2 and s_3 , along with a dimensionless coordinate σ will play significant roles similar to the coordinates in (2). The solution to (6) for this coordinate will be expressed as a Fourier series expansion in cosines:

$$\theta = \sum_n c_n \gamma_n(\tau) \cos \mu_n \sigma \quad (10)$$

where γ_n is a time function and the eigenvalues μ_n in the cosine argument are time dependent as the roots of the transcendental equation.

$$\mu_n t g \mu_n - \frac{al}{k(1 - A \cos \beta \tau)} = 0 \quad (11)$$

When we normalize the eigenfunctions, we get

$$c_n = \sqrt{2} \left(1 + \frac{\sin 2\mu_n}{\mu_n} \right)^{-\frac{1}{2}} \quad (12)$$

In one scenario, the dimensionless complex $al/k(1 - A \cos \beta \tau)$ in (11) is the analogue of the Biot criterion for a periodically deformed heat-carrying material. We obtain the following assessment from the data taking into account the time dependence of the component in the final expression is $Bi(t) \sim 0.03 - 1$. The time dependency causes "smearing" of the interval's border points. The roots of (12) vary insignificantly as a result of this "smearing," allowing us to treat. In the steady-state regime of muscular contractions, the heat-production of the skeletal muscle S_m may be assumed to be constant in the absence of muscle tremor under thermally comfortable conditions.

Function definition

$$G(\tau) = g\tau + \left[\left(1 + \frac{A^2}{2} \right) \tau + \left(2 \frac{A}{\beta} \right) \left(\sin \beta \tau - \left(\frac{A}{6} \right) - \sin 2\beta \tau \right) \right] \quad (13)$$

We get (10)'s solution in the form

$$\theta = \sum_n \frac{c_n^2 \sin \mu_n^2}{\mu_n} \cos \mu_n \sigma e^{-G(\tau)} \left\{ \left(1 - \frac{T_a}{T_0} \right) + \left[g \left(1 - \frac{T_a}{T_0} \right) + q \right] \times \int e^{G(\tau)} d\tau \right\} \quad (14)$$

According to the evaluations, the quantity $\beta \sim 10^4 / 10^5$. This number rises as the frequency of contractions rises. As a result, we get

$$G(\tau) \sim \tau \left[g + \mu_n^2 \left(1 + \frac{A^2}{2} \right) \right] \quad (15)$$

And the more forceful the muscular movement, the more accurate the last manifestation. We obtain by transforming (15)

$$T = T_a + (T_0 - T_a) e^{-G(\tau)} \sum_n \frac{c_n^2 \sin \mu_n^2}{\mu_n} \cos \mu_n \sigma \times \exp \left[-\mu_n^2 \tau \left(\left(1 + \frac{A^2}{2} \right) \right) \right] + \left[T_0 \left(1 + \frac{q}{g} - T_a \right) \right] \sum_n \frac{c_n^2 \sin \mu_n^2}{\mu_n} \cos \mu_n \sigma \times \frac{1}{1 + \frac{\mu_n^2}{g} \left(1 + \frac{A^2}{2} \right)} \quad (16)$$

These equations interpretation of the parameters governing heat transfer in skeletal muscle highlights their distinct roles. The contraction frequency (β) affects the rate of heat propagation, with higher contraction rates promoting faster heat spread due to more dynamic tissue deformation. The amplitude (A) of contractions influences the extent of geometric changes, where stronger contractions result in more pronounced effects on the local temperature distribution. The perfusion parameter (g) reflects the influence of blood flow, which plays a crucial role in maintaining long-term thermal equilibrium and preventing tissue overheating. Finally, the Biot number indicates the relative importance

of surface cooling versus internal conduction, helping to determine whether convective heat loss at the skin or internal thermal conduction dominates the temperature profile within the muscle.

RESULTS AND DISCUSSION:

The three-dimensional Figure (2) illustrates the dimensionless temperature distribution (θ) along the thickness of the skeletal muscle (σ) over dimensionless time (τ) during periodic contractions. The initial condition of the system corresponds to a uniform temperature profile, with θ approximately 0.189, representing the relative difference between arterial blood and ambient temperatures. As the simulation progresses, a clear temperature gradient develops along the muscle thickness.

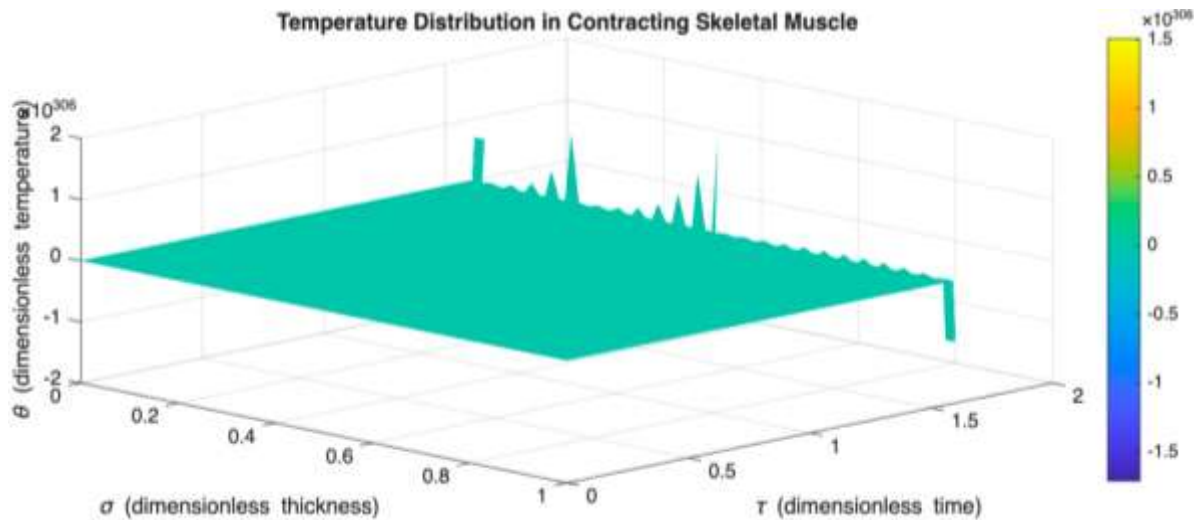


Figure (2): Temperature Distribution in Contracting Skeletal Muscle

The innermost layers of the muscle ($\sigma \approx 0$) exhibit higher temperatures, primarily due to metabolic heat generation (S_m) and perfusion effects, whereas the outer layers closer to the skin ($\sigma \approx 1$) show a lower temperature, reflecting heat loss through convective boundary conditions at the surface. This gradient confirms the significant role of blood perfusion and skin convection in regulating heat distribution during muscle activity. The periodic contractions of the muscle, introduce subtle oscillations in the temperature profile. Although these oscillations are small in magnitude, they indicate that dynamic muscle motion contributes to temporal variation in local tissue temperature, consistent with the Lagrangian treatment of moving boundaries [12]. The amplitude of oscillations would be expected to increase for higher contraction amplitudes A or higher frequencies β highlighting the importance of mechanical deformation on thermal behavior. The overall trend toward higher temperatures in the inner muscle region and gradual cooling toward the surface demonstrates the interplay between metabolic heat production, perfusion, and boundary heat loss. The results also show that the system approaches a quasi-steady-state over the simulated time interval, where temperature fluctuations due to contractions are relatively minor compared to the baseline thermal gradient. These findings are consistent with physiological expectations: skeletal muscle generates heat during activity, and the distribution of temperature is non-uniform, with maximum warming in deeper regions. The results further validate the use of Pennes' bioheat equation coupled with a moving-boundary Lagrangian framework to capture the effects of periodic muscle contractions on temperature distribution.

CONCLUSION

In conclusion, this study demonstrates that skeletal muscle exhibits significant spatiotemporal temperature variations during periodic contractions, governed by the interplay of metabolic heat generation, blood perfusion, and surface heat dissipation. Coupled thermo-mechanical modeling using Pennes' bioheat equation within a Lagrangian framework accurately captures these dynamics, revealing that deeper muscle regions attain higher temperatures and that periodic contractions introduce oscillatory thermal behavior. These findings highlight the critical role of mechanical activity in muscle thermoregulation and provide a robust computational foundation for applications in therapeutic interventions, sports performance optimization, and biomedical engineering. Future studies incorporating heterogeneous tissue

properties, vascular network dynamics, and pathological conditions could further enhance the predictive capability of skeletal muscle thermal modeling.

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